New Criteria for Exchange of a Spin-Zero Meson

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The Treiman-Yang test can be extended to give a new criterion for exchange of a spin-zero particle (SZE). The new test would be useful in a reaction (e.g., $\pi^- + p \to \rho + N^*$) in which the two final systems both have a detectable polarization. For such a case it would require that the polarizations of the two systems be uncorrelated, i.e., the cross section is a product of two factors, the first depending only on the polarization of one of the final systems and the second, only on the other. The analogous statements hold for correlations connecting one of the initial particles and the final system that communicates with it only through SZE, and also for correlations connecting the two initial particles. The new test would fail in the event that more than one spin-zero particle is exchanged, i.e., the collision amplitude is a superposition of several SZE amplitudes. In that case there could be correlation terms consisting of a scalar defined in the center-of-mass-frame (c.m.) of one of the final systems (with the momentum transfer as a preferred axis) multiplied by a scalar defined in the other final c.m. If the exchanged particles do not all have the same parity, then there may be additional terms containing a pseudoscalar in the first c.m. multiplied by a pseudoscalar in the second c.m. These effects do not violate the Treiman-Yang azimuthal-angle-independence and bombardment-energy-independence tests. Furthermore, the failure of the new criterion is only partial, since a multiple SZE model continues to forbid correlations of polarization components perpendicular to the momentum transfer in their respective frames of definition.

IT is possible to exploit the complexity of final states in reactions now being observed to make refined tests of models depending on exchange of a spin-zero meson, such as the one-pion exchange model for the process $\pi + N \to N^* + \rho$.

To see this, consider the process $A + B \to C + D$, where A and B are two colliding particles, and C and D are two (elementary or composite) systems produced by the collision. If we assume that for some values of the squared momentum transfer $t = (p_A - p_C)^2 = (p_B - p_D)^2$ the reaction is dominated by exchange of a spin-zero particle (SZE), then we may write the collision amplitude in the form

$$\mathcal{A}_{A+B\to C+D} = F_{\mathrm{I}} F_{\mathrm{II}} \quad \text{(SZE)}. \tag{I}$$

Here F_{I} is an invariant function which can be written in the center-of-mass frame (c.m.) of system C as a function only of t, of the angles θ with the momentum transfer, and the angles with each other of any polarization vectors $\mathbf{P}_{Ai}$, $\mathbf{P}_{Ci}$ of A and C, and finally of any undetermined energies E_{Ci} of the particles which compose C; F_{II} is defined similarly in terms of B and D (see Fig. 1). The factoring in Eq. (I) merely expresses the fact that a spin-zero particle cannot carry information coupling the system which (virtually) emits it and the system which absorbs it.

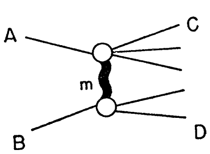

FIG. 1. Diagram for the contribution to the collision amplitude from the exchange of a single spin-zero meson m.

Treiman and Yang[1] pointed out two implications of the above statement. First, if we go to the c.m. of system $C(D)$ and take as an axis the momentum in that frame of $A(B)$, then no property of $C(D)$ may depend on the azimuthal angle ϕ_C (ϕ_D) about that axis.[2] Secondly, the amplitude $\mathcal{A}(t; \mathbf{P}_{Ai}, \mathrm{P}_{Ci}; E_{Ci}; \mathbf{P}_{Bi}, \mathbf{P}_{Di}, E_{Di})$ does not depend on the total energy W for the reaction, and therefore the cross section has only a trivial kinematic dependence on W.[3]

These two results do not exhaust the implications of Relation I. It also implies that for given W and t there must be no correlation of any polarization of system C, measured in the C c.m., with a polarization of system D, measured in the D c.m. The analogous statements hold for correlations between C and B, D and A, and for dependence of the cross section on the relative orientation of A and B.

This correlation test could have many applications. For example, if we assume that the reaction

$$\pi^- + p \to \rho^0 + N^* \to (\pi^+ + \pi^-) + (\pi^- + p)$$

can be described by one-pion exchange (OPE), then the angular distribution of the N^* decay with respect to the initial p direction, measured in the N^* c.m., should be unchanged if we consider separately those events in which the final π^+ went forward with respect to the initial π^- (as seen in the ρ c.m.) and those events in

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[1] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

[2] Tests of the dependence on the Treiman-Yang angle have been made for many reactions, especially $\pi + N \to \rho + N$. For this reaction a dependence on ϕ_{TY} was found at an incident pion momentum of 1.38 BeV/c in the laboratory [E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **9**, 170; 242(E) (1962)]. No dependence on ϕ_{TY} was found at 1.59 BeV/c [Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **24**, 515 (1963)] or 3.0 BeV/c [V. Hagopian and W. Selove, Phys. Rev. Letters **10**, 533 (1963)].

[3] This has been tested for $\pi + N \to \rho + N$ with fairly good agreement [Alfred S. Goldhaber, Phys. Rev. **134**, B600 (1964)].

which the π^+ went backward. Another example is the process

$$\pi^- + p \to \rho^0 + N.$$

If the initial proton is polarized, there must be no correlation of the direction of the ρ decay with the direction of proton polarization, if OPE applies. Similar comments apply to possible one-kaon-exchange (OKE) processes like

$$\pi^- + p \to K^* + \Lambda,$$

where correlation of the Λ and K^* decay directions might be observed if OKE did not describe the reaction. The main disadvantage of this test is that it requires one of A and C *and* one of B and D to have an easily measured polarization, but data for such processes are becoming available now. Furthermore, there exist models for such reactions which violate the SZE assumption and fail this test, but pass the test of Treiman and Yang for fixed W.[4] Even in a model which violates SZE, for example by permitting spin-one exchange, this kind of test provides a useful additional check because only certain correlations are possible if the particular model is right.[5] A further evident statement which should be kept in mind is that if two processes

$$A + B \to C + D$$
$$A + B' \to C + D'$$

are described by exchange of the same spin-zero particle, then the angular distribution of decay of C, measured in its c.m. frame, should be the same for both reactions, at a given t.

It is obvious that the requirement of no correlations of the sort discussed above is violated by many models, for example, a model permitting higher spin exchange. However, I wish to draw attention to a special class of violations which preserve the Treiman-Yang results, namely, the exchange of more than one spin-zero meson (multiple SZE, or MSZE). If the amplitude is the sum of several terms each coming from the exchange of a different spin-zero meson m,[6] then it may be written

$$\mathcal{A} = \sum_m F_{m\mathrm{I}} F_{m\mathrm{II}} \quad \text{(MSZE)}, \qquad \text{(II)}$$

where $F_{m\mathrm{I}}$ is an invariant formed from A and C, and $F_{m\mathrm{II}}$, from B and D. The resulting cross section will no longer factor into a quantity σ_C depending only on angles in the C c.m., multiplied by a similar form σ_D defined in the D c.m. Instead, the cross section may have several terms, each with a different $\sigma_C\sigma_D$, resulting in correlations forbidden by single SZE exchange. If the exchanged mesons do not all have the same parity, then there could also be correlation terms containing a pseudoscalar defined in the C c.m. multiplied by a pseudoscalar defined in the D c.m.

The MSZE cross section would of course not violate either of the two criteria of Treiman and Yang. In fact, not only is the cross section independent of the Treiman-Yang angle $\phi_C(\phi_D)$ of any vector in the $C(D)$ c.m., but also there is no correlation of ϕ_C with ϕ_D.

In order to clarify the concepts presented here, let me exhibit a single process which should be experimentally observable, and which includes all the possibilities described above. The process in question is $\bar{p} + p \to \bar{Y}^* + Y^*$. I assume the Y^* has spin $\frac{3}{2}^+$ and decays to Λ and π. The Λ polarization is given by its subsequent weak decay. The amplitude hypothesized is a sum of contributions from exchange of a pseudoscalar K, a hypothetical scalar K' (coupled to the S wave, not the P-wave, in the Λ-π system), and finally a vector K^* meson.[6a] The amplitude is

$$\mathcal{A} = \alpha_K \tilde{Y}^{\mu *} \Delta_\mu p \tilde{\bar{p}} \Delta^\nu \bar{Y}_\nu^* + \alpha_{K'} \tilde{\Lambda} \gamma^5 p \tilde{\bar{p}} \gamma^5 \bar{\Lambda} + \alpha_{K^*} \tilde{Y}_\mu^* \gamma^5 p g^{\mu\nu} \tilde{\bar{p}} \gamma^5 \bar{Y}_\nu^*.$$

The resulting cross section is proportional to

$$\sigma = |\alpha_K|^2 f_1 \bar{f}_1 + |\alpha_{K'}|^2 k f_2 \bar{f}_2 + 2 P_\Lambda P_{\bar{\Lambda}} \mathrm{Re}\,\alpha_K \alpha_{K'}^* k' f_3 \bar{f}_3 + |\alpha_{K^*}| k'' [|\alpha_{K^*}| (\gamma f_4 \bar{f}_4 + f_5 \bar{f}_5 + f_6 \bar{f}_6)^2 + \text{other terms}],$$

where $f_1 = 1 + 3\cos^2\theta$, $f_2 = 1$, $f_3 = 2\cos\theta\cos\theta_s - \sin\theta\sin\theta_s \times \cos(\phi - \phi_s)$, $f_4 = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos\phi$, $f_5 = \cos\theta \times \sin\theta_0 - \sin\theta\cos\theta_0\cos\phi$, $f_6 = \sin\theta\sin\phi$; k, k', k'' are kinematic factors; $P_\Lambda(P_{\bar{\Lambda}})$ gives the degree of polarization of $\Lambda(\bar{\Lambda})$; $\gamma = (1 - v^2)^{-1/2}$, where v is the $\bar{Y}^*$ velocity in the Y^* c.m.; θ is the angle, measured in the Y^* c.m., of the outgoing Λ momentum with respect to the incoming proton momentum; ϕ is the azimuth of the Λ about the proton momentum; θ_s, ϕ_s are the analogous quantities for the polarization of the Λ; θ_0 is the angle of the momentum of the $\bar{Y}^*$ to the incident proton momentum. The angles in the $\bar{Y}^*$ c.m. are defined by obvious analogy. The $\bar{f}$'s have the same dependence on the $\bar{Y}^*$ angles as do the corresponding f's on the corresponding Y^* angles.

By inspection, σ yields no correlations of $\bar{\theta}$ and θ if $\alpha_{K'}$ and α_{K^*} are zero. If $\alpha_{K'}$ and α_K are both nonzero, the quadratic terms give correlations of $\bar{\theta}$ and θ, while the interference term correlates pseudoscalars in the two c.m.'s.[7] Among the many terms resulting from the vector coupling if α_{K^*} is nonzero, the one shown already

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[4] For example, the K^* exchange model for $p + \bar{p} \to \Lambda + \bar{\Lambda}$: D. Bessis, C. Itzykson, and M. Jacob, Nuovo Cimento **27**, 376 (1963).

[5] I have analyzed the implications of general integer or half-integer spin J exchange. The only qualitative distinctions between the general case and SZE lie in the possibility of correlations between vectors perpendicular to the momentum transfer in the I(AC) and II(BD) systems with a total power $\mathcal{P} \leq 2J$ of dot products of the form $\mathbf{V}_\mathrm{I} \cdot \mathbf{V}_\mathrm{II}$, an energy dependence given by a $2J$-degree polynomial in W^2, and violations of azimuthal isotropy permitting a total power up to $2J$ in $\cos\phi_{\mathrm{TYI}}$ as well as $\cos\phi_{\mathrm{TYII}}$.

[6] From the point of view of dispersion relations, such a form would be the result of summing several pole terms, but the same effect could result from an integral over a cut.

[6a] For simplicity, I include only one of the linearly independent couplings of (pY^*) to $(\pi\rho)$ through the virtual K^*.

[7] Note that f_3 depends on the *relative* azimuthal angle of two vectors in the C c.m., but *not* on their *absolute* azimuthal angles.

exhibits the ϕ correlations, and ϕ dependence in each c.m. separately, which no SZE or MSZE process can produce.

We may conclude that an analysis of polarization correlations can distinguish three cases:

(1) Exchange of one spin-zero meson (SZE): No correlations of C and D polarizations are possible.

(2) Multiple spin-zero meson exchange (MSZE): There may be correlations between components of polarization along the momentum transfer direction in the C c.m. with components of polarization along the momentum transfer direction in the D c.m.

(3) Higher spin exchange: Violations of the two SZE criteria of Treiman and Yang, as well as correlations of the Treiman-Yang angles of directions in the two c.m.'s, may occur.

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Vector Harmonics for Three Identical Fermions*

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Orthonormal vector harmonics for the three-nucleon system are presented.

INTRODUCTION

The wave function of a system consisting of three nucleons depends on the coordinates τ_i, σ_i, $\mathbf{r}_i$, $i=1, 2, 3$, where τ_i is the two-valued isospin coordinate, σ_i is the two-valued spin coordinate, and the space coordinate $\mathbf{r}_i$ ranges over three-dimensional Euclidian space. The possible wave functions $\psi(\tau_i,\sigma_i,\mathbf{r}_i)$ are classified according to their transformation properties under translations, rotations, and reflections of the coordinate system and under permutations of the particle coordinates. The functions that belong to a definite representation $\mathbf{K}$ of the translation group are

$$\psi_{\mathbf{K}}=\exp(i\mathbf{K}\cdot\mathbf{R})\psi(\boldsymbol{\lambda},\boldsymbol{\rho},\sigma_i,\tau_i)\,,\quad i=1,\,2,\,3 \tag{1}$$

where

$$\begin{aligned}\mathbf{R}&=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)\,,\\ \boldsymbol{\lambda}&=6^{-1/2}(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)\,,\\ \boldsymbol{\rho}&=2^{-1/2}(\mathbf{r}_1-\mathbf{r}_2)\,.\end{aligned} \tag{2}$$

The vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ are invariant under translations. The internal wave functions ψ are chosen to have definite parity and to belong to definite irreducible representations of the quantum-mechanical rotation group SU_2, and the permutation group $S_3^{\tau\sigma r}$ where the Pauli principle specifies that ψ must belong to the antisymmetric representation of $S_3^{\tau\sigma r}$. The problem that will be considered here is the construction and parametrization of the functions ψ.

A function ψ that belongs to the antisymmetric representation of $S_3^{\tau\sigma r}$ can be split into parts that belong to definite irreducible representations of S_3^{τ}, S_3^{σ}, and S_3^{r} separately, where S_3^{τ}, S_3^{σ}, and S_3^{r} are the groups consisting of permutations of isospin, spin, and space coordinates only, respectively. Since these groups do not leave the Hamiltonian invariant, the three-nucleon wave function cannot belong to a single irreducible representation of one of these groups, but must be a linear combination of functions, each of which belongs to a single irreducible representation of each of the groups S_3^{τ}, S_3^{σ}, S_3^{r}. Since the irreducible representations of S_3^{σ} have definite total spin, the rotational classification is completed by requiring that the space part of the function have definite orbital angular momentum L and definite parity, besides belonging to a definite irreducible representation of S_3^{r}.

THE GROUP S_3 AND ITS REPRESENTATIONS

The group S_3 has three irreducible representations: a one-dimensional symmetric representation R_S, a one-dimensional antisymmetric representation R_A, and a two-dimensional mixed representation R_M. If φ belongs to R_S, then $P^i\varphi=\varphi$, where P^i is any permutation in S_3. Similarly, if φ belongs to R_A, then $P^i\varphi=\epsilon_i\varphi$ where $\epsilon_i=\pm1$ is the sign of the permutation P^i. If φ_1

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